SAFE HANDS & IIT-ian's PACE

MAJOR TEST-02 (JEE) ANS KEY Dt. 02-11-2023

PHYSICS		CHEMISTRY		MATHS	
Q. NO.	[ANS]	Q. NO.	[ANS]	Q. NO.	[ANS]
1	B	31	B	61	A
2	D	32	B	62	A
3	D	33	D	63	D
4	D	34	D	64	D
5	A	35	A	65	A
6	D	36	B	66	D
7	D	37	A	67	B
8	D	38	B	68	C
9	C	39	C	69	B
10	A	40	B	70	B
11	D	41	A	71	C
12	D	42	A	72	B
13	C	43	A	73	C
14	A	44	B	74	B
15	C	45	B	75	A
16	D	46	B	76	D
17	D	47	A	77	A
18	A	48	B	78	D
19	C	49	D	79	A
20	A	50	C	80	D
21	5	51	4	81	60
22	2	52	4	82	3
23	0.5	53	25	83	3
24	2	54	1	84	129
25	5	55	0	85	3
26	8	56	2	86	4
27	2	57	3	87	8
28	4	58	2	88	39
29	7	59	4	89	5
30	36	60	3	90	6

: ANSWER KEY :

1)	b	2)	d	3)	d	4)	d	21)	5	22)	2	23)	0.5	24)	2
5)	a	6)	d	7)	d	8)	d	25)	5	26)	8	27)	2	28)	4
9)	c	10)	a	11)	d	12)	d	29)	7	30)	36				
13)	c	14)	a	15)	c	16)	d								
17)	d	18)	a	19)	c	20)	a								

Single Correct Answer Type

1 **(b)**

$T = mg + \frac{mv^2}{l} = mg + \frac{m}{l}[2gl(1 - \cos\theta)]$

$= mg + 2mg(1 - \cos 60°) = 2mg = 2 \times 0.1 \times 9.8$

$= 1.96N$

2 **(d)**

For critical condition at the highest point $\omega = \sqrt{g/R}$

$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{R/g} = 2 \times 3.14\sqrt{4/9.8} = 4$ sec

3 **(d)**

Let u be initial velocity of projection at angle θ with the horizontal. Then, horizontal range,

$R = \frac{u^2 \sin 2\theta}{g}$

and maximum height $H = \frac{u^2 \sin 2\theta}{2g}$

Given, $R = 4\sqrt{3}H$

$\therefore \frac{u^2 \sin 2\theta}{g} = 4\sqrt{3} \cdot \frac{u^2 \sin^2\theta}{2g}$

$\therefore 2\sin\theta\cos\theta = 2\sqrt{3}\sin^2\theta$

or $\frac{\cos\theta}{\sin\theta} = \sqrt{3}$

or $\cot\theta = \sqrt{3} = \cot 30°$

4 **(d)**

$R = 4H\cot\theta$, if $R = 4\sqrt{3}H$ then $\cot\theta = \sqrt{3} \Rightarrow \theta = 30°$

5 **(a)**

As truck is moving on an incline plane therefore only component of weight ($mg\sin\theta$) will oppose the upward motion

Power = force × velocity = $mg\sin\theta \times v$

$= 30000 \times 10 \times \left(\frac{1}{100}\right) \times \frac{30 \times 5}{18} = 25\ kW$

6 **(d)**

$P = F\ v = (ma)v = m\left(\frac{d^2x}{dt^2}\right)\left(\frac{dx}{dt}\right)$

Since, power is constant

$\left(\frac{d^2x}{dt^2}\right)\left(\frac{dx}{dt}\right) = k$

or $\frac{d}{dt}\left(\frac{dx}{dt}\right)^2 = k$

$\left(\frac{dx}{dt}\right)^2 = k_1 t$

$\frac{dx}{dt} = \sqrt{k_1 t}$

$\frac{dx}{dt} = k_2(t)^{1/2} \quad (\therefore k_1^{1/2} = k_2)$

$x = k_3 t^{3/2} \left(\because k_3 = \frac{2}{3}k_2\right)$

Hence $\frac{dx}{dt} \propto t^{1/2} \propto x^{1/3}$

7 **(d)**

$P = \frac{mgh}{t}$

$m = \frac{Pt}{gh} = \frac{200 \times 60}{10 \times 10} = 1200$ L

8 **(d)**

Kinetic energy of particle ,$k = \frac{p_1^2}{2m}$

$p_1^2 = 2mk'$

When kinetic energy =2k

$p_2^2 = 2m \times 2k, p_2^2 = 2p_1^2, p_2 = \sqrt{2p_1}$

9 **(c)**

Since acceleration due to gravity is independent of mass, hence time is also independent of mass (or density) of object

10 **(a)**

The slope of distance-time graphs speed.

The change in velocity in 1 s

$= \tan 60° - \tan 45° = \sqrt{3} - 1$

$\therefore$ Acceleration $= \frac{\Delta v}{\Delta t} = \frac{\sqrt{3}-1}{1} = (\sqrt{3} - 1)$unit

11 **(d)**

In the positive region the velocity decreases linearly (during rise) and in the negative region velocity increases linearly (during fall) and the direction is opposite to each other during rise and fall, hence fall is shown in the negative region

12 **(d)**

The area under acceleration-time graph gives change in velocity.

13 **(c)**

Maximum force by surface when friction works

$F = \sqrt{f^2 + R^2} = \sqrt{(\mu R)^2 + R^2} = R\sqrt{\mu^2 + 1}$

Minimum force = R where there is no friction

Hence ranging from R to $R\sqrt{\mu^2 + 1}$

We get, $Mg \le F \le Mg\sqrt{\mu^2 + 1}$

14 **(a)**

$F = m\left(\frac{v - u}{t}\right) = \frac{5(65 - 15) \times 10^{-2}}{0.2} = 12.5\ N$

15 **(c)**

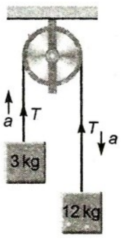
Acceleration of the car $= \frac{\text{Force on the car}}{\text{Mass of the car}}$

$= \frac{mnv}{M} = \frac{0.01 \times 10 \times 500}{2000}\text{ms}^{-2} = \frac{5}{200}\text{ms}^{-2}$

$= \frac{1}{40}\text{ms}^{-2}$

16 **(d)**

Equations of motion are
$m_1 a = T - m_1 g$
and $m_2 a = m_2 g - T$
$\Rightarrow \; 8a = T - 8g \quad \ldots\text{(i)}$
$12a = 12g - T \quad \ldots\text{(ii)}$
Form Eqs. (i) and (ii), we get
$a = \frac{g}{5} = 2\ \text{m/s}^2$
Substituting the value of a in Eq.(i)
We get $T = 96$ N



17 **(d)**

$F \propto v \Rightarrow F = kv \Rightarrow [k] = \left[\frac{F}{v}\right] = \left[\frac{MLT^2}{LT^{-1}}\right]$
$= [MT^{-1}]$

18 **(a)**
$X = [M^a L^b T^c]$

Maximum % error in $X = a\alpha + b\beta + c\gamma$

20 **(a)**
Energy $U = \frac{1}{2}LI^2$

$\Rightarrow \quad L = \frac{2U}{I^2}$

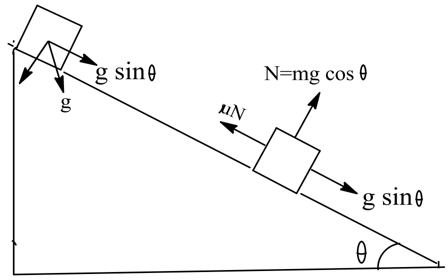
$\therefore [L] = \frac{[U]}{[I]^2} = \frac{[\text{ML}^2\text{T}^{-2}]}{[\text{A}]^2} = [\text{ML}^2\text{T}^{-2}\text{A}^{-2}]$

Integer Answer Type

21 **(5)**
$v_x = \frac{dx}{dt} = 3$ and $v_y = \frac{dy}{dt} = 4 - 10t = 4 - 10(0) = 4$
$v = \left[v_x^2 + v_y^2\right]^{1/2} = [3^2 + 4^2]^{1/2} = 5$ m/sec

22 **(2)**
$y = u \sin\theta t - \frac{1}{2}gt^2$
$15 = 52 \times \frac{5}{13}t - \frac{1}{2} \times 10t^2$
$5t^2 + -20t + 15 = 0$
$t^2 - 4t + 3 = 0$
$t^2 - 3t \times t + 3 = 0 \Rightarrow t(t-3) - 1(t-3) = 0$
$t_1 = 1$ s $\Rightarrow t_2 = 3$ s
Thus, $t_2 - t_1 = 3 - 1 = 2$ s

23 **(0.5)**



As, $v^2 - u^2 = 2as$
In the journey over the upper hall of incline,
$v^2 - 0 = 2(g \sin\theta)\frac{s}{2}$
$v^2 = g \sin\theta\, s$
In the journey over the lower hall of incline,
$v^2 - u^2 = 2\,as$
$0 - g\,s \sin\theta = 2\,g\,(\sin\theta - \mu\cos\theta)\frac{s}{2}$
$-\sin\theta = \sin\theta - \mu\cos\theta$
$\therefore \mu = \frac{2\sin\theta}{\cos\theta}$
$\therefore \mu = 2\tan\theta$
$= 2\tan 14^0$
$\therefore \mu = 2 \times 0.25$
$\therefore \mu = 0.5$

24 **(2)**
Maximum elongation is given by
$x_{\max} = \frac{2[F_1 m_2 + F_2 m_1]}{K(m_1 + m_2)}$
Here $F_1 = F_2; m_1 = m$ and $m_2 = M$
Put the values and solve to get $x_{\max} = 2F/K$

25 **(5)**
$s = u + \frac{a}{2}(2n - 1)$
$u = 100\ \text{ms}^{-1}, a = -10\ \text{ms}^{-2}$ and $s = 5$ m
$5 = 100 - 5(2n - 1)$ gives $n = 10$ s
Body when thrown up with velocity 200 ms^{-1} will take 20 s to reach the highest point. Distance travelled in 20th second is $200 - 5(200 \times 2 - 1) = 5$ m
In the last second of upward journey, the bodies will travel same distance

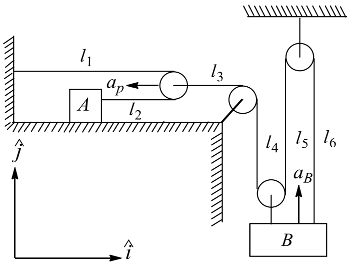
26 **(8)**
$t_1 = t_2 - t, v_1 = v_2 = v, S = \frac{1}{2}a_1 t_2^1, S = \frac{1}{2}a_2 t_2^2$
$v_1 = a_1 t_1, v_2 = a_2 t_2 \Rightarrow v_2 + v = a_1 t_1$
$\Rightarrow a_2 t_2 + v = a_1 t_1 = a_1 t_2 \Rightarrow t_2 = \frac{v + a_1 t}{a_1 - a_2}$
$\sqrt{\frac{a_2}{a_1}} = \frac{t_1}{t_2} = 1 - \frac{t}{t_2} \Rightarrow \sqrt{\frac{a_2}{a_1}} = 1 - \frac{t(a_1 - a_2)}{(v + a_1 t)}$
$\Rightarrow \frac{\sqrt{a_2}}{\sqrt{a_1}} = \frac{v + a_2 t}{v + a_1 t} \Rightarrow \sqrt{a_2}\,v + a_1\sqrt{a_2}\,t$
$= v\sqrt{a_1} + a_2\sqrt{a_1}t$

$\Rightarrow v = (\sqrt{a_1 a_2})t = 8\ \text{ms}^{-1}$

27 **(2)**

$\ell_1 + \ell_2 = C \Rightarrow \ell_1' + \ell_2' = 0$

$\Rightarrow -a_p + (12 - a_p) = 0 \Rightarrow a_p = 6\ ms^{-2}$



$\ell_3 + \ell_4 + \ell_5 + \ell_6 = C \Rightarrow \ell_3' + \ell_4' + \ell_5' + \ell_6' = 0$

$a_P - a_B - a_B - a_B = 0 \Rightarrow a_P = 3a_B \Rightarrow a_B = 2\ \text{ms}^{-1}$

28 **(4)**

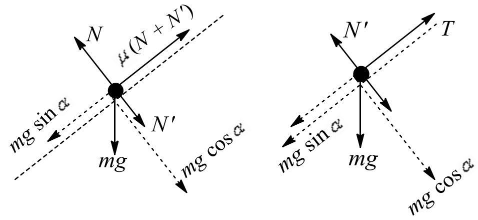
Since A tends to slip down, frictional forces act on it from both sides up the plane

Let N be the reaction of the plank on A and N' be the mutual normal action-reaction between A and B

From the free-body diagram of A

$N' + mg\cos\alpha = N$ and $mg\sin\alpha = \mu(N + N')$

From the free-body diagram of B



$N6'' = mg\cos\alpha$

$mg\sin\alpha + \mu N' = T$

$\therefore\ 2\,mg\cos\alpha = N$

and $mg\sin\alpha = \mu(2\,mg\cos\alpha + mg\cos\alpha)$

or $\mu = \frac{1}{3}\tan\alpha = \frac{1}{3} \times \frac{3}{4} = 0.25$ or 1/m=4

29 **(7)**

Dimension of force = $[M^1L^1T^2]$

Now, F = ma

$\Rightarrow m = \frac{f}{a}$

$\therefore$ Mass $= \frac{10^3 N}{10^{-2} m\ (10^1 s)^{-2}} = \frac{10^3 kg\ m}{s^2} \times \frac{10^2 s^2}{10^{-2} m}$

$= 10^7 kg$

Comparing 10^x with 10^7,

$\therefore x = 7$

30 **(36)**

The dimensional formula of force is $[M^1L^1T^1]$ 1000 $[M_2^1 L_2^1 T_2^{-2}]$

$\therefore\ x = 1000\left[\frac{g}{1000\ g}\right]^1 \left[\frac{cm}{100\ cm}\right]^1 \left[\frac{sec}{60\ sec}\right]^{-2}$

$= \frac{1000 \times 3600}{10^5}$

$= 36$ units

MATHS ANS KEY & SOLUTIONS

: ANSWER KEY :

61)	a	62)	a	63)	d	64)	d
65)	a	66)	d	67)	b	68)	c
69)	b	70)	b	71)	c	72)	b
73)	c	74)	b	75)	a	76)	d
77)	a	78)	d	79)	a	80)	d
81)	60	82)	3	83)	3	84)	129
85)	3	86)	4	87)	8	88)	39
89)	5	90)	6				

: HINTS AND SOLUTIONS :

Single Correct Answer Type

62 **(a)**

We have,

$\tan\theta+\tan\left(\theta+\frac{3\pi}{4}\right)=2$

$\Rightarrow \tan\theta+\frac{\tan\theta-1}{1+\tan\theta}=2$

$\Rightarrow \tan^2\theta+2\tan\theta-1=2+2\tan\theta$

$\Rightarrow \tan^2\theta=3$

$\Rightarrow \tan^2\theta=\tan^2\frac{\pi}{3}\Rightarrow \theta=n\pi\pm\frac{\pi}{3}, n\in Z$

64 **(d)**

$\frac{\sin 55°-\cos 55°}{\sin 10°}=\frac{\sin 55°-\sin 35°}{\sin 10°}$

$=\frac{2\cos 45°.\sin 10°}{\sin 10°}$

$=\sqrt{2}$

65 **(a)**

We have, $(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+..$

If x is replace by $-\left(1-\frac{1}{x}\right)$ and n is $-n$, then expression

becomes $\left[1-\left(1-\frac{1}{x}\right)\right]^{-n}$

$=1+(-n)\left[-\left(1-\frac{1}{x}\right)\right]+\frac{(-n)(-n-1)}{2!}\left[-\left(1-\frac{1}{x}\right)\right]^2+\ldots$

$\Longrightarrow x^n=1+n\left(1-\frac{1}{x}\right)+\frac{n(n+1)}{2!}\left(1-\frac{1}{x}\right)^2+\ldots$

66 **(d)**

Let $f=\left(8-3\sqrt{7}\right)^{10}$, here $0<f<1$

$\therefore$ $\left(8+3\sqrt{7}\right)^{10}+\left(8-3\sqrt{7}\right)^{10}$ is an integer hence, this is the value of n

67 **(b)**

Since, total number of terms $=59+1=60$

$\therefore$ Required sum $=\frac{2^{59}}{2}=2^{58}$

68 **(c)**

Let

$S=(1+x)^{1000}+2x(1+x)^{999}+3x^2(1+x)^{998}+\cdots+1000x^{999}(1+x)+1001x^{1000}$ ... (i)

$\therefore \frac{x}{1+x}S=x(1+x)^{999}+2x^2(1+x)^{998}+\cdots+1000x^{1000}+\frac{1001x^{1001}}{1+x}$ ... (ii)

Subtracting (ii) from (i), we get

$\left(1-\frac{x}{x+1}\right)S=(1+x)^{1000}+x(1+x)^{999}+x^2(1+x)^{998}+\cdots+x^{1000}-\frac{1001x^{1001}}{1+x}$

$\Rightarrow S=(1+x)^{1001}+x(1+x)^{1000}+x^2(1+x)^{999}+\cdots+x^{1000}(1+x)-1001x^{1001}$

$\Rightarrow S=(1+x)^{1001}\frac{\left\{1-\left(\frac{x}{1+x}\right)^{1001}\right\}}{\left\{1-\frac{x}{1+x}\right\}}-1001x^{1001}$

$\Rightarrow S=(1+x)^{1002}\left\{1-\left(\frac{x}{1+x}\right)^{1001}\right\}-1001x^{1001}$

$\Rightarrow S=(1+x)^{1002}-x^{1001}(1+x)-1001x^{1001}$

$\therefore$ Coefficient of x^{50} in S is ${}^{1002}C_{50}$

69 **(b)**

We have,

$2^{\sin^2 x}\cdot 3^{\cos^2 y}\cdot 4^{\sin^2 z}\cdot 5^{\cos^2\omega}\geq 120$

$\Rightarrow 2^{\sin^2 x}\cdot 3^{\cos^2 y}\cdot 4^{\sin^2 z}\cdot 5^{\cos^2\omega}\geq 2\times 3\times 4\times 5$

$\Rightarrow \sin^2 x\log 2+\cos^2 y\log_3+\sin^2 z\log 4+\cos^2\omega\log 5\geq \log 2+\log 3+\log 4+\log 5$

$\Rightarrow \cos^2 x\log 2+\sin^2 y\log 3+\cos^2 z\log 4+\sin^2\omega\log 5\leq 0$

$\Rightarrow \cos^2 x=0, \sin^2 y=0, \cos^2 z=0$ and $\sin^2\omega=0$

$\Rightarrow x=m\pi\pm\frac{\pi}{2}, m\in Z; y=n\pi, n\in Z$

$z=r\pi\pm\frac{\pi}{2}, r\in Z;\ \omega=t\pi, t\in Z$

But, $x,y,z,\omega\in[0,10]$

$\therefore x=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2}, y=0,\pi,2\pi,3\pi, z=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2}$

and $\omega=0,\pi,2\pi,3\pi$

Hence, the number of ordered 4-tuples is $3\times 4\times 3\times 4=144$

70 **(b)**

We have, $e^x=x(x+1), x<0$

Consider the curves $y=e^x$ and $y=x(x+1)$ for $x<0$. Graphs of these two curve intersect at exactly one point. So, the equation $e^x=x(x+1)$ has exactly one real root

71 **(c)**

Let

$f(x)=x^8-x^5-\frac{1}{x}+\frac{1}{x^4}=\frac{x^{12}-x^9-x^3+1}{x^4}$

$=\frac{(x^9-1)(x^3-1)}{x^4}$

Clearly, $f(x) \geq 0$ for all $x < 0$ and it is not defined for $x = 0$
For $0 < x < 1$, we have
$x^9 - 1 < 0$ and $x^3 - 1 < 0 \Rightarrow f(x) > 0$
For $x \geq 1$, we have $x^9 - 1 \geq 0$ and $x^3 - 1 \geq 0 \Rightarrow f(x) \geq 0$
Hence, $f(x) \geq 0$ for all $x \neq 0$

73 **(c)**
We have,
$(2.3)^x = (0.23)^y = 1000$
$\Rightarrow 2.3 = 10^{3/x}$ and $0.23 = 10^{3/y}$
$\Rightarrow 2.3 = 10^{3/x}$ and $2.3 = 10^{3/y+1}$
$\Rightarrow \log_{10} 2.3 = \frac{3}{x}$ and $\log_{10} 2.3 = \frac{3}{y} + 1$
$\Rightarrow \frac{3}{x} - \frac{3}{y} = 1 \Rightarrow \frac{1}{x} - \frac{1}{y} = \frac{1}{3}$

74 **(b)**
Since, $x_1, x_2, \dots, x_n$ are any real numbers.
$\therefore \quad \frac{x_1^2 + x_2^2 + \dots + x_n^2}{n} \geq \left(\frac{x_1 + x_2 + \dots + x_n}{n}\right)$ [using mth power theorem]
$$\Rightarrow \quad n \sum_{i=1}^{n} x_i^2 \geq \left(\sum_{i=1}^{n} x_i\right)^2$$

75 **(a)**
Since, $2b = a + c$
Now, $(a + 2b - c)(2b + c - a)(a + 2b + c)$
$= (a + a + c - c)(a + c + c - a)(2b + 2b)$
$= 2a.2c.4b = 16abc$

76 **(d)**
Given series, is a arithmetic geometric series.

Here, $a_1 = 1,\ d = 1,\ r = a$

$$\therefore S_\infty = \frac{a_1}{1 - r} + \frac{d.r}{(1 - r)^2}$$

$$= \frac{1}{1 - a} + \frac{1.a}{(1 - a)^2} = \frac{1}{(1 - a)^2}$$

77 **(a)**
Let α, β, γ are the roots of the given equation.
Then,
$\alpha + \beta + \gamma = -p$
$\alpha\beta + \beta\gamma + \gamma\alpha = -q$
And $\alpha\beta\gamma = -r$
Now, $pq = (\alpha + \beta + \gamma)(\alpha\beta + \beta\gamma + \gamma\alpha)$
$= (0 + \gamma)[\alpha\beta + \gamma(\alpha + \beta)]$ ($\because \alpha + \beta = 0$ is given)
$= \alpha\beta\gamma$
$= -r$

78 **(d)**
Given equation is $e^{\sin x} - e^{-\sin x} - 4 = 0$
Let $e^{\sin x} = y$, then given equation can be written as
$y^2 - 4y - 1 = 0 \Rightarrow y = 2 \pm \sqrt{5}$
But the value of $y = e^{\sin x}$ is always positive so we take only $y = 2 + \sqrt{5}$
$\Rightarrow \log_e y = \log_e(2 + \sqrt{5})$
$\Rightarrow \sin x = \log_e(2 + \sqrt{5}) > 1$
Which is impossible since $\sin x$ cannot be greater than 1.
Hence, we cannot find any real value of x which satisfies each given equation.

79 **(a)**
Let $z = \sqrt{3} + i$
$\therefore \quad \arg(z) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$
For making a right angled ΔOPQ, point Q either in IInd quadrant or IVth quadrant
If the point Q is in IInd quadrant, then we take
$\theta = 120°$
$\therefore \tan 120° = -\cot 30° = \frac{\sqrt{3}}{-1}$
$\therefore$ Point Q is $(-1, \sqrt{3})$ and if the point Q is in IVth quadrant then we take
$\theta = -60°$
$\therefore \quad \tan(-60°) = -\tan 60° = -\frac{1}{\sqrt{3}}$
$\therefore$ Point Q is $(1, \sqrt{3})$

80 **(d)**
Since, α and β are the roots of
$\lambda x^2 + (1 - \lambda)x + 5 = 0$
$\therefore \alpha + \beta = \frac{\lambda - 1}{\lambda}, \alpha\beta = \frac{5}{\lambda}$
Since, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{4}{5}$
$\Rightarrow \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{4}{5}$
$\Rightarrow \frac{(\lambda - 1)^2 - 10\lambda}{5\lambda} = \frac{4}{5}$
$\Rightarrow \lambda^2 - 16\lambda + 1 = 0$
Now, $\lambda_1 + \lambda_2 = 16$ and $\lambda_1.\lambda_2 = 1$
$\therefore \frac{\lambda_1}{\lambda_2^2} + \frac{\lambda_2}{\lambda_1^2} = \frac{\lambda_1^3 + \lambda_2^3}{(\lambda_1\lambda_2)^2}$
$= \frac{(\lambda_1 + \lambda_2)^3 - 3\lambda_1\lambda_2(\lambda_1 + \lambda_2)}{(1)^2}$
$= (16)^3 - 3 \times 1(16)$
$= 4048$

Integer Answer Type

81 **(60)**

$\cos^{-1}x = a \Rightarrow x = \cos a, \quad 0 < a < \frac{\pi}{4}$

$\sin^{-1}\left(2x\sqrt{1-x^2}\right) + \sec^{-1}\left(\frac{1}{2x^2-1}\right) = \frac{2\pi}{3}$

$\Rightarrow \sin^{-1}(2\sin a\cos a) + \sec^{-1}\left(\frac{1}{2\cos^2 a - 1}\right) = \frac{2\pi}{3}$

$\Rightarrow \sin^{-1}(\sin 2a) + \sec^{-1}(\sec 2a) = \frac{2\pi}{3}$

$\Rightarrow 4a = \frac{2\pi}{3} \qquad \ldots\left(0 < 2a < \frac{\pi}{2}\right)$

$\Rightarrow a = \frac{\pi}{6}$

$\Leftrightarrow \cos^{-1}x = \frac{\pi}{6} \Rightarrow x = \frac{\sqrt{3}}{2}$

$\Rightarrow \tan^{-1}(2x) = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$ radians

$= 60$ (in degrees)

82 **(3)**

$\log_2 15 \,.\, \log_{1/6} 2 \,.\, \log_3 1/6$

$= \frac{\log 15}{\log 2} . \frac{\log 2}{-\log 6} . \frac{-\log 6}{\log 3}$

$= \log_3 15$

83 **(3)**

$(1 + 0.00002)^{50000} = \left(1 + \frac{1}{50000}\right)^{50000}$

Now we know that $2 \le \left(1 + \frac{1}{n}\right)^n < 3 \ \forall\, n \ge 1 \Rightarrow$ Least integer is 3

84 **(129)**

$T_{r+1} = \ {}^{1024}C_r \left(5^{\frac{1}{2}}\right)^{1024-r} \left(7^{\frac{1}{1}}\right)^{r}$

For an integral term, both $\frac{1024-r}{2}$ and $\frac{r}{8}$ must be non-negative integers.

$\Rightarrow r = 0, 8, 16, \ldots, 1024$

$\Rightarrow$ Number of integral terms $= 129$

85 **(3)**

$a + b = 3$

HM $\le$ AM for 3 numbers $\frac{a}{2}, \frac{a}{2}, b$ we have

$\frac{3}{\frac{2}{a} + \frac{2}{a} + \frac{1}{b}} \le \frac{\frac{a}{2} + \frac{a}{2} + b}{3} = 1;$

$\therefore\ 1 \ge \frac{3}{\frac{2}{a} + \frac{2}{a} + \frac{1}{b}} \Rightarrow \frac{2}{a} + \frac{2}{a} + \frac{1}{b} \ge 3$

$\therefore\ \frac{4}{a} + \frac{1}{b} \ge 3$

86 **(4)**

We have $\frac{2\left(\frac{x}{2}\right)+y}{3} \ge \left(\left(\frac{x}{2}\right)^2 y\right)^{1/3}$

$\Rightarrow \left(\frac{3}{3}\right)^3 \ge \frac{x^2 y}{4} \Rightarrow x^2 y \le 4$

Therefore, maximum value of x^2y is 4

87 **(8)**

For the G.P. $a, ar, ar^2, \ldots$

$P_n = a(ar)(ar^2)\ldots(ar^{n-1}) = a^n . r^{n(n-1)/2}$

$\therefore S = \sum_{n=1}^{\infty} \sqrt[n]{P_n} = \sum_{n=1}^{\infty} ar^{(n-1)/2}$

Now, $\sum_{n=1}^{\infty} ar^{(n-1)/2} = a\left[1 + \sqrt{r} + r + r\sqrt{r} + \ldots + \infty\right] = \frac{a}{1-\sqrt{r}}$

Given $a = 16$ and $r = 1/4$

$\therefore S = \frac{16}{1-(1/2)} = 32$

88 **(39)**

Let

$S_n = 1 + 6 + \frac{9(1^2+2^2+3^2)}{7} + \frac{12(1^2+2^2+3^2+4^2)}{9} + \cdots$

$= \frac{3(1^2)}{3} + \frac{6(1^2+2^2)}{5} + \frac{9(1^2+2^2+3^2)}{7} + \cdots T_n$

$= \frac{(3n)\left[\frac{n(n+1)(2n+1)}{6}\right]}{2n+1} = \frac{n^3+n^2}{2} S_n$

$= \sum T_n = \sum\left(\frac{n^3+n^2}{2}\right) = \frac{1}{2}(\sum n^3 + \sum n^2)$

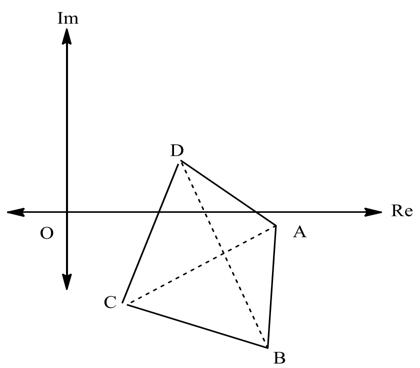
$= \frac{1}{2}\left[\left(\frac{n(n+1)}{2}\right)^2 + \frac{n(n+1)(2n+1)}{6}\right]$

$= \frac{n(n+1)}{4}\left[\frac{n(n+1)}{2} + \frac{2n+1}{3}\right] S_{12}$

$= \frac{12(13)}{4}\left[\frac{12(13)}{2} + \frac{2(12)+1}{3}\right] = 39\left(\frac{259}{3}\right)$

$\Rightarrow m = 39$

89 **(5)**



Vector MD represents $1 + i - (2 - i) = -1 + 2i$

vector MA represents $= \frac{1}{2}[i(-1+2i)] \ldots \left[|MA| = \frac{1}{2}|DM|\right]$

$|MA| = \sqrt{\frac{1}{4}+1} = \frac{\sqrt{5}}{2} \Rightarrow |AC| = \sqrt{5}$

$|DM| = \sqrt{(-1)^2 + 2^2} = \sqrt{5}$

$\Rightarrow |BD| = 2\sqrt{5}$

$\Rightarrow$ Area of [] ABCD

$= \frac{1}{2}|AC||BD|$

i.e. S $= \frac{1}{2}(\sqrt{5})(2\sqrt{5})$

$\Leftrightarrow [S] = 5$

90 **(6)**

We have $|a\omega + b| = 1$

$\Rightarrow |a\omega + b|^2 = 1$

$\Rightarrow (a\overline{\omega} + b) = 1$

$\Rightarrow a^2 + ab(\omega + \overline{\omega}) + b^2 = 1$

$\Rightarrow a^2 - ab + b^2 = 1$

$\Rightarrow (a-b)^2 + ab = 1 \quad (1)$

When $(a-b)^2 = 0$ and $ab = 1$ then $(1, 1); (-1, -1)$

When $(a-b)^2 = 1$ and $ab = 0$ then $(0, 1); (1, 0); (0, -1); (-1, 0)$

Hence there are 6 ordered pairs